L.G.Philippenko

On the hypothesis about of the "continuous structure" of shock waves in a continuous bodies

I. In the proposed work the material objects (the "bodies") are considered as a continuous ones*. A such model is natural for the physical bodies which are formed by a condensed materials: solid, liquid, and the dance gases. The wave functions of its forming corpuscles essentially covers again, and inner motions in such materials has an expressed collective character. For the rare gases the such model is a less suitable one: here the more preferable is that which is used in the molecular - kinetic theories of the matter.
The any continuous body $\mathfrak{B}$ can be separated (let by mental) - in general by arbitrary manner - on the sections $\Delta\mathfrak{B}$, everyone of which also is a some continuous body, suitable for further the same dividing (and in the same reasons). When the all dimensions of section $\Delta\mathfrak{B}$ are enough small*, it will for the brevity be colled as "the particle"*. The transferences of the particles in a space are determined by their velocities ("the mass velocities") relatively to some systems of the count, among which without fail is present (had been stipulated obviously or by passing over on silence) the inertial - "immovable" - system of the count with had been defined in it the orthogonal Descartes coordinate system $OXYZ$ ("the initial system of coordinate"). The mass velocity in the initial coordinate system will be marked as $\vec{U}$, but in all others - as $\vec{\upsilon}$.
By consider the processes in the inner sections of the body $\mathfrak{B}$, far from its boundaries, during of the time intervals in which the possible changes beside the boundaries of $\mathfrak{B}$ will not have time to visibly influence on the conditions in the inner regions, the body $\mathfrak{B}$ we can (approximately) imagine as an infinitely extended. Here for a simplicity will be investigated the inner motions of the material in an infinitely extended body $\mathfrak{B}$, which in the initial coordinate system are characterized by field of velocities

$$\vec{U} \uparrow\uparrow \vec{e}_X \;, \qquad U = U\left(X, t\right) \;; \qquad\qquad (I.1)$$

just such motions are mostly considered in the contemporary hydrodynamic theory of shock waves, in particular when it is discussed the problem of its "continuous structure".

In an existent literature the general interpretation of the term "shock wave" is not quoted. For example, in [2] (see chapter IX, paragraph 85) we read: "We will do one the terminology remark. Under the term "shock wave" we understand the rupture surface itself. In the literature, however, it can meet an another terminology, in which the rupture surface is called "the shock wave front", and under the shock wave there understand the rupture surface in common with a followed behind it a continuous motion of the gas". For all that "The ruptures of continuity in a gas motion have place along any surfaces; by passing across such a surface the indicated values** undergo a leaps. This surfaces are called the surfaces of rupture". [2] (see chapter IX, paragraph 84).None the less, "…the shock waves with the little over leaps of the values are in reality the

* The strict definition of the notions "the continuous body", "the particle", "the enough small value" and others see in [1].
** There it bear in mind the characterizations of motion of the matter: the velocity, the density, the pressure, and so on.

transitional layers with the not nought thickness *, which decrease by the increase of the size of leaps."[2] (see chapter IX, paragraph 93).
In this conditions, here is taken in consideration the following definition of notion "the shock wave" ("SW")**:

(*) *As the shock wave here is called the section of motion of the matter* h:

$$h(t) = X_{(+)}(t) - X_{(-)}(t), \qquad h \geq 0, \tag{I.2}$$

the dynamical conditions of compatibility on which - that is conditions on the its boundary planes $X_{(-)}$ *and* $X_{(+)}$ *- satisfy to demands of the four postulates of the motion: the conservation of the mass, the impuls and the energy , and the second principle of thermodynamics*****, with any values of pressures on this planes; the breadth of the section* h *is acknowledged as decreasing with an increase of the amplitude of SW.*
When the motion in SW is a stationary one, h no depend from t.
The finding of the answer on the question, is the SW containing without fail the rupture surface, or it is in a reality "… the transitional layer /of the continuous motion/ with a not nought thickness", forms the topic of the proposed work. As regards to the terms "the rupture surface" and "the shock front", here it understand as synonyms.
The character of motion of matter on the h in the case $h > 0$ - it is just so called "continuous structure" of SW.

II. Thus, it is scrutinized the one - dimensional (with a field of velocities I.1) motion of matter in the inner sections of the highly large (with an enough exactness - of the "infinite large") continuous bogy $\mathfrak{B}$, which contains the shock wave on the moving along the axis X section h. The motion of matter in front of SW , at $X > X_{(+)}(t)$, and behind of it, at $X < X_{(-)}(t)$, is supposed as the continuous one (in a most of the published work about the "structure" of SW the motion in this regions is supposed even constant).
Let us consider the motion of the matter in the SW with the "continuous structure" – that is the motion on the $h > 0$ satisfying the definition (*). In the neighbourhood of the boundaries $X_{(+)}$ and $X_{(-)}$ its profile by any parameter of motion - for instance, by the specific volume V - can be presented by the decomposition in the Taylor series:

$$V_t(X) = V_t\left(X_{(\pm)}\right) + \left(X - X_{(\pm)}\right)^k \left(d^k V_t / dX^k\right)_{X=X_{(\pm)}} / k! \tag{II.1}$$

In order to the declaration "on the section h it maintains the shock wave" would have even though some sense, the motion of the matter on this section must even though be something differs from its motion in front and behind of the SW (where a "shock wave" is absent); "in a tongue" of the expression (1) it means that even though some from the derivatives $d^k V_t / dX^k$ at $X = X_{(\pm)}$, had been derived from the left and to the right, must differs from one another. The planes $X = X_{(\pm)}(t)$, on which some derivatives of the profile suffers the rupture, but profile itself remains continuous, are called as characteristics and move along the matter everyone with a local velocity of sound c [2]:

---

* As it is obviously from the next text in [2], the motion of the material in the limits of this layers is considered as continuous.

** It is quoted for application with a had been taken into consideration the field of velocities (I.1)

*** Not any others postulates, suppositions and assumptions, besides of the four indicated, no consider.

$$dX_{(\pm)}(t)/dt \equiv \dot{X}_{(\pm)}(t) = c_{(\pm)} \tag{II.2}$$

But $\dot{X}_{(+)}$ - the velocity along axis X of the front boundary of the section h - according to (2) is the velocity of SW along the matter in front of it; in a contemporary theory it marks as D, and the sound velocity in front of SW - as c_0. The equation (2) signifies, that the velocity along the matter of SW, possessing the continuous "structure", under any other conditions (different materials, amplitudes of wave, et al.) satisfies to the demand:

$$D \equiv \dot{X}_{(+)} = c_0 \tag{II.3}$$

and the velocity of the behind boundary of such SW - to the demand:

$$U_{(-)} \equiv \dot{X}_{(-)} = c_{(-)} \equiv c \tag{II.4}$$

Because of (3), not any shock wave, which velocity exceeds the sound velocity in a matter in front of it, can not possess the continuous "structure". It is worth to note, that just the SW with a $D > c_0$ form the highest interest for the practical applications, and , as a rule, just by this sign the motions usual are qualified as the SW among the all types of wave motions.
As regards to the wave motions satisfying to demands (3) and (4), - those are the usual continuous waves of deformation. In a contemporary hydrodynamics theory of SW, as an energy equation, it always is used the Hugoniot equation, which is just only if absent the heat exchange between the matter embraced in a process "the shock wave" and the rest of material in front and behind of it (see also n^0 III). For the motion on the section of "continuous structure" $h > 0$, it signifies the adiabatic character of deformations on its boundary planes $X = X_{(\pm)}(t)$. Therefore in all so far published works about of the "continuous structure" of SW, in the equations (3) and (4) under c_0 and c it must bear in mind the adiabatic sound velocities in a matter in front and behind of wave. But it means that this waves itself (settled down on the section $h > 0$) are the well known longitudinal waves in a linear elastic material. Thus, the "continuous structure" on $h > 0$ can possess only such wave motions, the deformation in which take place in the limits of elasticity of material*: then, as it is known, the profile of the wave on section h can possess any form determined by D'Alembert solution of the problem of theory of elasticity with had been indicated in it the boundary conditions.

But if $h = 0$, that is

$$X_{(-)}(t) = X_s(t), \qquad X_{(+)}(t) = X_s(t) + 0, \tag{II.5}$$

and on this $V(X_s) \neq V(X_s + 0)$, then the SW contains the rupture plane $X = X_s(t)$, and decomposition (1) in its vicinity at $X > X_s$ can to contain only derivatives to the right, and at $X < X_s$ - only from the left; the plane $X = X_s(t)$ already is not a characterization plane and the demands (3) and (4) are unacceptable: the velocity of SW, $D \equiv \dot{X}_s$, is not obliged to be equel to sound velocity, and the deformations of material - to the elastic ones.
All here told by no means was bound with a value of the "amplitude" of SW, that is with a difference between the strains $(P_{xx})_t$ on the boundaries $X = X_{(\pm)}(t)$ for the wave with a

---

* It is so by condition that as the energy equation in a dynamical conditions of compatibility it is used the Hugoniot equation - so as it just have place in all so far published works about of the "continuous structure" of SW.

"structure", or with a value of the leap $\left[\left(P_{xx}\right)_t\right]$ on the plane $X = X_s(t)$ for the SW represented by the rupture plane. Therefore all told above conserve with any amplitudes of wave.

The more broad and detail discussion of the hypothesis about of "continuous structure" of SW and its inner contradictions had been executed in [1].

III. In the works declaring the existence of "continuous structure" in the shock waves* , that hypothesis is bound (obviously or by silent) with that experimental fact that in most practically interested cases the specific entropy of the material s changes in a shock wave. The change of entropy is determined by the second principle of thermodynamics [3]:

$$ds = d_e s + d_i s,$$
$$d_e s = dQ/T, \qquad d_i s \geq 0; \tag{III.1}$$

here $d_e s$ - the result of outer no - mechanical influences on the particle, dQ - the elementary change of its specific inner energy at the expense of such influences ("influx of the heat"), T - the temperature, and $d_i s$ - the result of the inner no reversible processes in the particle**. To the nymber of no reversible processes in a first turn concerns - besides of heat transference - the inner friction ("viscosity") of materials. Accordingly, in the equations of laws of conservation, on the basis of which must be described the motion of matter, in the cases when $\Delta s \neq 0$ it keeps first of all the members describing the influence of the heat transference and viscosity on the motion along the section h. But, as it is shown in the n^0 II, the SW can to have a continuous "structure" on $h > 0$ only if deformations here not exceeds the limits of a linear elasticity of material. Meanwhile, the elastic deformation, by it himself definition, is not compatible with a processes of dissipation of the energy***: if it presence, the body, after returning to initial not-deforming state, should possess the greater inner energy than at the beginning (before the deforming), while the surrounding bodies, had carried out its deformation - the less energy (quite clear definition of the appearance of elasticity see for example in [4,5,6]). All processes taken place in the bounds of elasticity of material are isoentropic: in it $\Delta s = 0$ .Accordingly, the dissipating members in the equations of laws of conservation, were intended for a description of the influence of heat exchange and viscosity on the motion of material, in a SW with a continuous "structure", at the correct its write-up, must be absent (it equals to nought identically); therefore are deprived of the physical sense all conclusions making by those or any manipulations with this members, as it have place in a diverse hydrodynamic theories of a "thickness of shock waves" in the model of continuous environment.

As one of fundamental thesis of contemporary hydrodynamic theory of SW is an adiabatic hypothesis: it assumes that the deformation of material in any SW (in any materials, at any amplitudes and so on) is carried out adiabatically - in a that sense that the elementary particle no changes by the heat with the material in front and behind of wave within a time of crossing of SW. Just therefore the equation of the conservation of energy on the SW acquires the form of known Hugoniot equation:

---

* Its existence in a such works always, in essence, is postulated: its existence declares "a priori", and all next reasonings are used for justification and explanation of this declaration.

** The physical sense, the analytical properties and peculiarity of the behaviour of functions s and Q by a position of the physical kinetics are examined in [1].

*** Just therefore in a linear elastic body the tie of stress and strain is a mutual simple: it no contain any other variable parameters of motion.

$$\varepsilon_{(-)} - \varepsilon_{(+)} = \tfrac{1}{2}\left(p_{(-)} + p_{(+)}\right)\left(V_{(+)} - V_{(-)}\right) ; \qquad \text{(III.2)}^{*}$$

if the specific inner energy on the plane $X_{(-)}(t)$, $\varepsilon_{(-)}$, to express over $V_{(-)}$ and $p_{(-)}$, and similarly to act with a $\varepsilon_{(+)}$, then the equation (2) will determine the line on the plane of states (p,V), which is called as "the shock adiabat" of material. Together with the equations of conservation mass and umpuls the equation (2) forms in a contemporary theory the basis system of equations (the Rankine - Hugoniot system), which is considered as the departure for analysis of any SW in the any conditions of their existence.

But on the SW - never mind, contain it the rupture plane or not, - the motion in which satisfy to Hugoniot equation, the specific entropy of material conserves: this fact in detail (analytically) is proved in [1,7,8] . If - as it have place in a contemporary theory of SW - the Hugoniot equation ascribe to the any SW, then in all those it must acknowledge invariability of entropy of matter whiles of it crossing. The reason of such unnatural conclusion (contradicting also to experimental facts) consist in what as a matter of fact the adiabatic hypothesis, and with it the Hugoniot equation, is just only in the limits of linear elastic deformations: outside of this limits this equation no satisfy to the energy conservation law and must be replaced on the equation taking into account the shock heat transference (shock heat exchange). The detailed proof of said is contained in the works [1,7,8].

Bibliography

1. Л.Г.Филиппенко. Сильные ударные волны в сплошных телах. - Киев, УМК ВО, 1992г. После опубликования монография существенно дополнена и доработана автором //L.G.Philippenko. Strong Shock Waves in the Continuous Bodies. – Kiev: UMK VO, 1992. After of the publication the monograph had been essentially completed and corrected by author//

2. Л.Д.Ландау, Е.М.Лифшиц. Гидродинимика. - М., "Наука", 1988г.

3. S.R.deGroot and P.Mazur. Non – Equilibrium thermodynamics. Nort – Holland Publishing Comp., Amsterdam 1962 // С. деГроот и П.Мазур. Неравновесная термодинамика. - М., "Мир", 1964г.//

4. Физический энциклопедический словарь, т.5, статья "Упругость." - М., Изд. "Советская энциклопедия, 1966г.

5. А.И.Лурье, "Теория упругости". - М., "Наука", 1970г.

6. Л.И.Седов. Механика сплошной среды, т.1. - М., "Наука", 1970г.

7. Philippenko L.G. The Consequences from the Hugoniot Equation. “Hydromechanics” - № 36. – Kiev: “Scientific Thought”, 1977. (ФилиппенкоЛ.Г. Следствия из уравнения Гюгонио. Сборник «Гидромеханика». – Вып. 36. – Киев: «Наукова думка», 1977). The English text of this article is placed at URL: https://secure.filesanywhere.com/main/xfolder_main.aspx

8. L.G.Philippenko. The Shock Heat Transference. . . . .

* At $h=0$ this equation have a form: $\varepsilon_s - \varepsilon_0 = \tfrac{1}{2}(p_s + p_0)(V_0 - V_s)$

L.G.Philippenko
07.08.2008